GRADE 4

Mathematics Assessment Anchors and Eligible content: Aligned to Pennsylvania Common Core Standards

The Assessment Anchors, as defined by the Eligible Content, are organized into cohesive blueprints, each

structured with a common labeling system that can be read like an outline. This framework is organized first by

Reporting Category, then by Assessment Anchor, followed by Anchor Descriptor, and then finally, at the greatest level of detail, by an Eligible Content statement. The common format of this outline is followed across the PSSA.

Here is a description of each level in the labeling system for the PSSA:

**Reporting Category**

The Assessment Anchors are organized into four classifications, as listed below.

•A = Numbers and Operations •C = Geometry

•B = Algebraic Concepts •D = Data Analysis and Probability

These four classifications are used throughout the grade levels. In addition to these classifications, there are five Reporting Categories for each grade level. The first letter of each Reporting Category represents the classification; the second letter represents the Domain as stated in the Common Core State Standards for Mathematics. Listed below are the Reporting Categories for Grade 3.

•A-T = Number and Operations in Base Ten

•A-F = Number and Operations - Fractions

•B-O = Operations and Algebraic Thinking

•C-G = Geometry

•D-M = Measurement and Data

The title of each Reporting Category is consistent with the title of the corresponding Domain in the Common Core State Standards for Mathematics. The Reporting Category title appears at the top of each page.

**Assessment Anchor**

The Assessment Anchor appears in the shaded bar across the top of each Assessment Anchor table. The

Assessment Anchors represent categories of subject matter (skills and concepts) that anchor the content of the PSSA. Each Assessment Anchor is part of a Reporting Category and has one or more Anchor Descriptors unified under and aligned to it.

**Anchor Descriptor**

Below each Assessment Anchor is one or more specific Anchor Descriptors. The Anchor Descriptor adds a level of specificity to the content covered by the Assessment Anchor. Each Anchor Descriptor is part of an Assessment Anchor and has one or more Eligible Content unified under and aligned to it.

**Eligible Content**

The column to the right of the Anchor Descriptor contains the Eligible Content statements. The Eligible

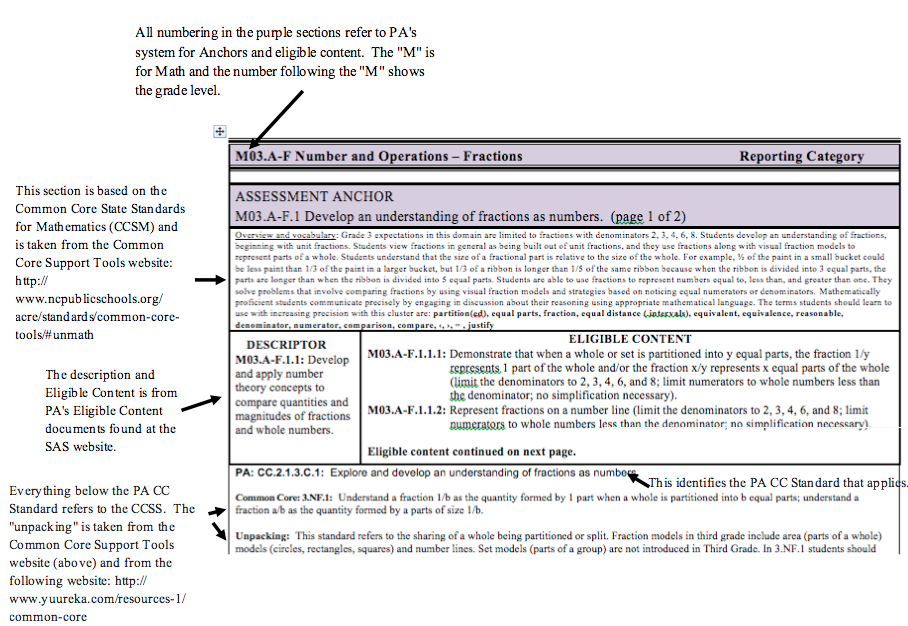
Content is the most specific description of the skills and concepts assessed on the PSSA. This level is considered the assessment limit and helps educators identify the range of the content covered on the PSSA. **Note:** All Grade 3 Eligible Content is considered Non-Calculator.

**Reference**

In the space below each Assessment Anchor table is a code representing one or more Common Core State Standards for Mathematics that correlate to the Eligible Content statements.

Alignment to the “National” Common Core and “unpacking” can be found below the PA CC Standard.

How Do I Read This Document?



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| **M04.A-T Number and Operations in Base Ten Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.A-T.1 Generalize place value understanding for multi-digit whole numbers. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.) | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, ‹, ›, =, comparisons/compare, round** | |
| **DESCRIPTOR**  **M04.A-T.1.1:** Apply place-value and numeration concepts to compare, find equivalencies, and round. | **ELIGIBLE CONTENT**  **M04.A-T.1.1.1:** Demonstrate an understanding that in a multi-digit whole number (through  1,000,000), a digit in one place represents ten times what it represents in the  place to its right. Example: Recognize that in the number 770, the 7 in the  hundreds place is ten times the 7 in the tens place.  **M04.A-T.1.1.2:** Read and write whole numbers in expanded, standard and word form  through 1,000,000.  **M04.A-T.1.1.3:** Compare two multi-digit numbers through 1,000,000 based on meanings of  the digits in each place, using >, =, and < symbols.  **M04.A-T.1.1.4:** Round multi-digit whole numbers (through 1,000,000) to any place. |
| **PA CC.2.1.4.B.1:** Apply place value concepts to show an understanding of multi-digit whole numbers.  **Common Core: 4.NBT.1:** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*  **Unpacking:** This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. Students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with. Example: How is the 2 in the number 582 similar to and different from the 2 in the number 528?  **Common Core: 4.NBT.2:** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.  **Unpacking:** This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones.  •Read and write multi-digit whole numbers and go back and forth between each of these forms:  • Base-10 numbers (e.g., 145);  • Number names (e.g., One-hundred forty-five);  • Expanded form (e.g., 145 = 1 × 100 + 4 × 10 + 5 × 1).  • Compare two multi-digit numbers and use >, =, and < symbols to record the results of comparisons.    **Common Core: 4.NBT.3:** Use place value understanding to round multi-digit whole numbers to any place.  **Unpacking:** This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.  Example 1: Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?  Student 1: *First, I multiplied 3 and 6 which equals* *18. Then I multiplied 6 and 6 which is* *36. I know 18 plus 36 is about 50. I’m trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.*  Student 2: *First, I multiplied 3 and 6 which* *equals 18. Then I multiplied 6 and 6* *which is 36. I know 18 is about 20* *and 36 is about 40. 40+20=60. 300-* *60 = 240, so we need about 240 more* *bottles.*  Example 2: On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel? Some typical estimation strategies for this problem:  Student 1: *I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.*  Student 2: *I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.*  Student 3: *I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.*  *•* Round any whole number to a given place by looking at the place to the right to decide whether to round up or round down. | |

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| **M04.A-T Number and Operations in Base Ten Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.A-T.2 Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.) | |
| Overview and vocabulary: Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **products, groups of, quotients, partitioned equally, multiplication, division, equal groups, arrays, patterns, equations, unknown, difference, subtraction, addition, sum, addend, factor** | |
| **DESCRIPTOR**  **M04.A-T.2.1:** Use operations to solve problems. | **ELIGIBLE CONTENT**  **M04.A-T.2.1.1:** Add and subtract multi-digit whole numbers (limit sums and subtrahends up to and  including 1,000,000).  **M04.A-T.2.1.2:** Multiply a whole number of up to four digits by a one-digit whole number and multiply 2  two-digit numbers.  **M04.A-T.2.1.3:** Divide up to four-digit dividends by one-digit divisors with answers written as whole-  number quotients and remainders.  **M04.A-T.2.1.4:** Estimate the answer to addition, subtraction, and multiplication problems using whole  numbers through six digits (for multiplication, no more than 2 digits x 1 digit, excluding  powers of 10). |
| **PA: CC.2.1.4.B.2:**  Use place value understanding and properties of operations to perform multi-digit arithmetic.  **Common Core: 4.NBT.4:** Fluently add and subtract multi-digit whole numbers using the standard algorithm.  **Unpacking:** Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract. This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property). This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use. When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works. Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.  •Add any whole numbers using the standard algorithm (i.e., stacking the numbers and then adding the ones column, “carrying the one” if necessary, then adding the tens column, etc.).  •Subtract any two whole numbers using the standard algorithm (i.e., stacking the numbers and subtracting the ones column (“borrowing” from the tens if necessary), then subtracting the tens column, etc.).  **Common Core: 4.NBT.5:** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.  **Unpacking:** Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5th grade.  **Common Core: 4.NBT.6:** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.  Unpacking: In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context. Example: A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box? • Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50. • Using Place Value: 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4) • Using Multiplication: 4 x 50 = 200, 4 x 10 = 40, 4 x 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65. This standard calls for students to explore division through various strategies. | |

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| **M04.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.A-F.1 Extend understanding of fraction equivalence and ordering. | |
| Overview and vocabulary: Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **partition(ed), fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, comparison/compare, ‹, ›, =, benchmark fraction** | |
| **DESCRIPTOR**  **M04.A-F.1.1:** Find equivalencies and compare fractions | **ELIGIBLE CONTENT**  **M04.A-F.1.1.1:** Recognize and generate equivalent fractions.  **M04.A-F.1.1.2:** Compare two fractions with different numerators and different denominators  (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100) using the symbols >, =, or <, and  justify the conclusions. |
| **PA: CC.2.1.4.C.1:**  Extend the understanding of fractions to show equivalence and ordering.  **Common Core: 4.NF.1**: Explain why a fraction **a**/**b** is equivalent to a fraction (**n** x **a**) / (**n** x **b**) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.  **Unpacking:** This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100). This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.  Example:  Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80    •Recognize when two fractions are equivalent or not equivalent.  •Explain and illustrate why two fractions are equivalent or not equivalent.  •Generate equivalent fractions for a given fraction.  **Common Core: 4.NF.2:** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.  **Unpacking:** •Determine if two fractions are equivalent; and  •If the two fractions are not equivalent, say which fraction is greater (or less) than the other by using such strategies as:  -Comparing fractions when the numerators are the same;  -Comparing fractions when the denominators are the same or by creating common denominators;  -Comparing fractions based on their size;  -Comparing fractions based on their location on a number line;  -Using visual fraction models.  •Record the result of the comparison using symbols >, =, or <, and justify the conclusions.  This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students’ experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (ie, ½ and 1/8 of two medium pizzas is very different from 1/2 of one medium and 1/8 of one large).  Example 1: Use patterns blocks. 1. If a red trapezoid is one whole, which block shows 1/3.? 2. If the blue rhombus is 1/3, which block shows one whole? 3. If the red trapezoid is one whole, which block shows 2/3?  Example 2: Mary used a 12 x 12 grid to represent 1 and Janet used a 10 x 10 grid to represent 1. Each girl shaded grid squares to show ¼. How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they need to shade different numbers of grid squares?  Possible solution: Mary shaded 36 grid squares; Janet shaded 25 grid squares. The total number of little squares is different in the two grids, so  ¼ of each total number is different.  Example 3: There are two cakes on the counter that are the same size. The first cake has ½ of it left. The second cake has 5/12 left. Which cake has more left? | |

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| **M04.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.A-F.2 Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (page 1 of 2) | |
| Overview and vocabulary: Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operations, addition/joining, subtraction/separating, fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, decomposing, mixed number,(properties)-rules about how numbers work, multiply, multiple** | |
| **DESCRIPTOR**  **M04.A-F.2.1:** Solve problems involving  fractions and whole numbers (straight computation or word problems).  These 3 EC addressed  on next page. | **ELIGIBLE CONTENT**  **M04.A-F.2.1.1:** Add and subtract fractions with a common denominator (denominators limited to 2, 3,  4, 5, 6, 8, 10, 12, and 100; answers do not need to be reduced; no improper fractions  as the final answer).  **M04.A-F.2.1.2:** Decompose a fraction or a mixed number into a sum of fractions with the same  denominator (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100), recording the  decomposition by an equation. Justify decompositions (for example, by using a visual  fraction model). Example 1: 3/8 = 1/8 + 1/8 + 1/8 OR 3/8 = 1/8 + 2/8  Example 2: 2 1/12 = 1 + 1 + 1/12 = 12/12 + 12/12 + 1/12  **M04.A-F.2.1.3:** Add and subtract mixed numbers with a common denominator (denominators limited  to 2, 3, 4, 5, 6, 8, 10, 12, and 100; no regrouping with subtraction; fractions do not  need to be reduced; no improper fractions as the final answers).  **M04.A-F.2.1.4:** Solve word problems involving addition and subtraction of fractions referring to the  same whole or set and having like denominators (denominators limited to 2, 3, 4, 5, 6,  8, 10, 12, and 100).  **M04.A-F.2.1.5:** Multiply a whole number by a unit fraction (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100; final  answers do not need to be reduced or written as a mixed number). Example: 5 x (1/4) = 5/4  **M04.A-F.2.1.6:** Multiply a whole number by a non-unit fraction (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100; final  answers do not need to be reduced or written as a mixed number). Example: 3 x (5/6) = 15/6  **M04.A-F.2.1.7:** Solve word problems involving multiplication of a whole number by a fraction (denominators limited to 2, 3, 4,  5, 6, 8, 10, 12, and 100). |
| **PA: CC.2.1.4.C.2:** Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.  **Common Core: 4.NF.3:** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.  a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.  b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.  Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 +*  *8/8 + 1/8.*  c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using  properties of operations and the relationship between addition and subtraction.  d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using  visual fraction models and equations to represent the problem.  **Unpacking: 4.NF.3.a-**A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of the same whole. Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.  **4.NF.3.b**-Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.  **4.NF.3.c-**A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions. Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers into improper fractions.  **4.NF.3.d-**A cake recipe calls for you to use ¾ cup of milk, ¼ cup of oil, and 2/4 cup of water. How much liquid was needed to make the cake?  THIS ANCHOR CONTINUED ON NEXT PAGE | |

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| **M04.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.A-F.2 Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (page 2 of 2) | |
| **DESCRIPTOR**  These 4 EC addressed  on pervious page.  **M04.A-F.2.1:** Solve problems involving  fractions and whole numbers (straight computation or word problems). | **ELIGIBLE CONTENT**  **M04.A-F.2.1.1:** Add and subtract fractions with a common denominator (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and  100; answers do not need to be reduced; no improper fractions as the final answer).  **M04.A-F.2.1.2:** Decompose a fraction or a mixed number into a sum of fractions with the same denominator (denominators  limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100), recording the decomposition by an equation. Justify decompositions  (for example, by using a visual fraction model). Example 1: 3/8 = 1/8 + 1/8 + 1/8 OR 3/8 = 1/8 + 2/8  Example 2: 2 1/12 = 1 + 1 + 1/12 = 12/12 + 12/12 + 1/12  **M04.A-F.2.1.3:** Add and subtract mixed numbers with a common denominator (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12,  and 100; no regrouping with subtraction; fractions do not need to be reduced; no improper fractions as the final  answers).  **M04.A-F.2.1.4:** Solve word problems involving addition and subtraction of fractions referring to the same whole or set and  having like denominators (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100).  **M04.A-F.2.1.5:** Multiply a whole number by a unit fraction (denominators limited to 2, 3, 4, 5, 6, 8,  10, 12, and 100; final answers do not need to be reduced or written as a mixed  number). Example: 5 x (1/4) = 5/4  **M04.A-F.2.1.6:** Multiply a whole number by a non-unit fraction (denominators limited to 2, 3, 4, 5, 6,  8, 10, 12, and 100; final answers do not need to be reduced or written as a mixed  number). Example: 3 x (5/6) = 15/6  **M04.A-F.2.1.7:** Solve word problems involving multiplication of a whole number by a fraction  (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100). |
| **PA: CC.2.1.4.C.2:** Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.  **Common Core: 4.NF.4**: Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.  a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the*  *conclusion by the equation 5/4 = 5 x (1/4).*  b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a*  *visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5. (In general, n x (a/b) = (n x a)/b.)*  c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent  the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many*  *pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*  **Unpacking: 4.NF.4.a-**This standard builds on students’ work of adding fractions and extending that work into multiplication.  Example: 3/6 = 1/6 + 1/6 + 1/6 = 3 x (1/6)    **4.NF.4.b-** This standard extended the idea of multiplication as repeated addition. For example, 3 x (2/5) = 2/5 + 2/5 + 2/5 = 6/5 = 6 x (1/5). Students are expected to use and create visual fraction models to multiply a whole number by a fraction.    **4.NF.4.c.-** This standard calls for students to use visual fraction models to solve word problems related to multiplying a whole number by a fraction.  Example: In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race?    Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns. | |

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| **M04.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.A-F.3 Understand decimal notation for fractions, and compare decimal fractions. | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, equivalent, reasoning, decimals, tenths, hundreds, multiplication, comparisons/compare, ‹, ›, =** | |
| **DESCRIPTOR**  **M04.A-F.3.1:** Use operations to solve problems involving decimals, including converting between fractions and decimals (may include word problems). | **ELIGIBLE CONTENT**  **M04.A-F.3.1.1:** Add two fractions with respective denominators 10 and 100. Example: Express  3/10 as 30/100, and add 3/10 + 4/100 = 30/100 + 4/100 = 34/100.  **M04.A-F.3.1.2:** Use decimal notation for fractions with denominators 10 or 100. Example:  Rewrite 0.62 as 62/100 and vice versa.  **M04.A-F.3.1.3:** Compare two decimals to hundredths using the symbols >, =, or <, and justify  the conclusions. |
| **PA: CC.2.1.4.C.3:** Connect decimal notation to fractions, and compare decimal fractions (base 10 denominator, e.g. 19/100).  **Common Core: 4.NF.5**: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)  **Unpacking: NOTE:** This standard is **NOT** about converting the fractions into decimals. Emphasis should be placed on naming the fractions properly; e.g., calling 7/10, “seven tenths”.  This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.  This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.  **Common Core: 4.NF.6:** Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.  **Unpacking:** Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal. Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.    Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100. Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value.    **Common Core: 4.NF.7:** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.  **Unpacking:** Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.    When the wholes are the same, the decimals or fractions can be compared. Example: Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths. | |

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| **M04.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.B-O.1 Use the four operations with whole numbers to solve problems. | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/divide, addition/add, subtraction/subtract, equations, unknown, remainders, reasonableness, mental computation, estimation, rounding** | |
| **DESCRIPTOR**  **M04.B-O.1.1:** Use numbers and symbols to model the concepts of expressions and equations. | **ELIGIBLE CONTENT**  **M04.B-O.1.1.1:** Interpret a multiplication equation as a comparison. Represent verbal statements of  multiplicative comparisons as multiplication equations. EX 1: Interpret 35 = 5 x 7 as a  statement that 35 is 5 times as many as 7 and 7 times as many as 5. EX 2: Know that the  statement 24 is 3 times as many as 8 can be represented by the equation 24=3x8 or 24 = 8x3.  **M04.B-O.1.1.2:** Multiply or divide to solve word problems involving multiplicative comparison, distinguishing  multiplicative comparison from additive comparison. EX: Know that 3 x 4 can be used to  represent that Student A has 4 objects and Student B has 3 times as many objects, and not just  3 more objects.  **M04.B-O.1.1.3:** Solve multi-step word problems posed with whole numbers using the four operations.  Answers will be either whole numbers or have remainders that must be interpreted  yielding a final answer that is a whole number. Represent these problems using equations  with a symbol or letter standing for the unknown quantity.  **M04.B-O.1.1.4:** Identify the missing symbol (+,–,x,÷,=,<,>) that makes a # sentence true (1-digit divisor only). |
| **PA:CC.2.2.4.A.1:** Represent and solve problems involving the four operations.  **Common Core: 4.OA.1:** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.  **Unpacking:** A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., “a is n times as much as b”). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.  Students should be given opportunities to write and identify equations and statements for multiplicative comparisons. Example: 5 x 8 = 40. Sally is five years old. Her mom is eight times older. How old is Sally’s Mom? 5 x 5 = 25 Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?  **Common Core: 4.OA.2**: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.  **Unpacking:** This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems. Examples: **Unknown Product:** A blue scarf costs $3. A red scarf costs 6 times as much. How much does the red scarf cost? (3 x 6 = p). **Group Size Unknown:** A book costs $18. That is 3 times more than a DVD. How much does a DVD cost?  (18 ÷ p = 3 or 3 x p = 18). **Number of Groups Unknown:** A red scarf costs $18. A blue scarf costs $6. How many times as much does the red scarf cost compared to the blue scarf? (18 ÷ 6 = p or 6 x p = 18).  • When distinguishing multiplicative comparison from additive comparison, students should note that  - additive comparisons focus on the difference between two quantities (e.g., Deb has 3 apples and Karen has 5 apples. How many more apples  does Karen have?). A simple way to remember this is, “How many more?”  - multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller  than the other (e.g., Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?). A simple way to remember  this is “How many times as much?” or “How many times as many?”  **Common Core: 4.OA.3:** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.  **Unpacking:** The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.  Example: On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total? A typical estimation strategy for this problem: **Student 1**: *I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.* The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.  This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders: • Remain as a left over • Partitioned into fractions or decimals • Discarded leaving only the whole number answer • Increase the whole number answer up one • Round to the nearest whole number for an approximate result | |

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| **M04.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.B-O.2 Gain familiarity with factors and multiples. | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/divide, factor pairs, factor, multiple, prime, composite** | |
| **DESCRIPTOR**  **M04.B-O.2.1:** Develop and apply number theory concepts to represent numbers in various ways. | **ELIGIBLE CONTENT**  **M04.B-O.2.1.1:** Find all factor pairs for a whole number in the interval 1 through  100. Recognize that a whole number is a multiple of each of its  factors. Determine whether a given whole number in the interval 1  through 100 is a multiple of a given one-digit number. Determine  whether a given whole number in the interval 1 through 100 is prime  or composite. |
| **PA:CC.2.2.4.A.2:** Develop and/or apply number theory concepts to find factors and multiples.  **Common Core: 4.OA.4:** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.  **Unpacking:** This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8. A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.  Prime vs. Composite: A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors.  Students investigate whether numbers are prime or composite by:  • building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into  only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number  • finding factors of the number  Students should understand the process of finding factor pairs so they can do this for any number 1 - 100, Example: Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.  Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).  Example:  Factors of 24: 1, 2, 3, 4, 6, 8,12, 24  Multiples of:  1: 1, 2, 3, 4, 5 . . .  2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 . . .  3: 3, 6, 9, 12, 15, 18, 21, 24 . . .  4: 4, 8, 12, 16, 20, 24 . . .  8: 8, 16, 24 . . .  12: 12, 24 . . .  To determine if a number between1-100 is a multiple of a given one-digit number, some helpful hints include the following:  • all even numbers are multiples of 2  • all even numbers that can be halved twice (with a whole number result) are multiples of 4  • all numbers ending in 0 or 5 are multiples of 5  Students should be able to:  •List all of the factor pairs for a whole number in the range 1-100.  •Use an organized strategy in order to be assured that all factor pairs are listed.  •Say whether or not a given whole number in range 1-100 is a multiple of a given one-digit number and defend the choice.  •Say whether a given whole number in the range 1-100 is prime, composite, or neither, and defend the choice. | |

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| **M04.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.B-O.3 Generate and analyze patterns. | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **pattern (number or shape), pattern rule** | |
| **DESCRIPTOR**  **M04.B-O.3.1:** Recognize, describe, extend, create, and replicate a variety of patterns. | **ELIGIBLE CONTENT**  **M04.B-O.3.1.1:** Generate a number or shape pattern that follows a given rule. Identify apparent  features of the pattern that were not explicit in the rule itself. Example 1: Given the  rule “Add 3” and the starting number 1, generate terms in the resulting sequence  and observe that the terms alternate between odd and even numbers.  Example 2: Given the rule “increase the number of sides by 1” and starting with a  triangle, observe that the tops of the shapes alternate between a side and a vertex.  **M04.B-O.3.1.2:** Determine the missing elements in a function table (limit to +, –, or x and to whole  numbers or money).  **M04.B-O.3.1.3:** Determine the rule for a function given a table (limit to +, –, or x and to whole  numbers). |
| **PA:CC.2.2.4.A.4:** Generate and analyze patterns using one rule.  **Common Core: 4.OA.5:** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*  **Unpacking:** Patterns involving numbers or symbols **either** repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations. Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.    After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.  Example: Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.  This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.  Example: There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days? | |

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| **M04.C-G Geometry Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.C-G.1 Draw and identify lines and angles, and classify shapes by properties of their lines and angles. | |
| Overview and vocabulary: Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **classify shapes/figures, (properties)-rules about how numbers work, point, line, line segment, ray, angle, vertex/vertices, right angle, acute, obtuse, perpendicular, parallel, right triangle, isosceles triangle, equilateral triangle, scalene triangle, line of symmetry, symmetric figures, two dimensional From previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere** | |
| **DESCRIPTOR**  **M04.C-G.1.1:** List properties, classify, draw and identify geometric figures in two dimensions. | **ELIGIBLE CONTENT**  **M04.C-G.1.1.1:** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and  parallel lines. Identify these in two-dimensional figures.  **M04.C-G.1.1.2:** Classify two-dimensional figures based on the presence or absence of parallel or  perpendicular lines, or the presence or absence of angles of a specified size. Recognize right  triangles as a category, and identify right triangles.  **M04.C-G.1.1.3:** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such  that the figure can be folded along the line into mirroring parts. Identify line-symmetric  figures and draw lines of symmetry (up to two lines of symmetry). |
| **PA:CC.2.3.4.A.1:** Draw lines and angles and identify these in two-dimensional figures.  **PA:CC.2.3.4.A.2:** Use the understanding of fractions to partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.  **PA:CC.2.3.4.A.3:** Recognize symmetric shapes and draw lines of symmetry.  **Common Core: 4.G.1:** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.  **Unpacking:** This standard asks students to draw two-dimensional geometric objects and to also identify them in two-dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines. Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students do not easily identify lines and rays because they are more abstract. They should be able to explain the difference between lines, line segments and rays. Students should be able to draw and identify right angles, acute angles, obtuse angles, straight angles, lines, line segments, parallel lines, perpendicular lines, and rays.  **Common Core: 4.G.2**: Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.  **Unpacking:** Angle Measurement: This expectation is closely connected to 4.MD.5, 4.MD.6, and 4.G.1. Students’ experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles. Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.  •Sort and classify two dimensional figures into four groups: •Sort triangles into three groups:  -Have parallel lines and perpendicular lines -Acute triangle  -Have parallel lines but no perpendicular lines -Obtuse triangle  -Have perpendicular lines but no parallel lines -Right triangle  -Have no parallel or perpendicular lines •Identify right triangles regardless of their orientation.  **Common Core: 4.G.3**: Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.  **Unpacking:** •Categorize two-dimensional figures as line-symmetric and non-line-symmetric.  •Draw in the line(s) of symmetry for line-symmetric shapes. •Explain why a given shape is non-line-symmetric.  Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry. This standard only includes line symmetry not rotational symmetry. Example: For each figure, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions. Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex. | |

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| **M04.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.D-M.1 Solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit. (Page 1 of 2) | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft),**  **yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter** | |
| **DESCRIPTOR**  **M04.D-M.1.1:** Solve problems involving length, weight (mass), liquid volume, time, area, and perimeter.  These 3 ED  addressed on next  page. | **ELIGIBLE CONTENT**  **M04.D-M.1.1.1:** Know relative sizes of measurement units within one system of units including standard  units (in., ft, yd, mi; oz., lb; c, pt, qt, gal), metric units (cm, m, km; g, kg; mL, L), and  time (sec, min, hr, day, wk, mo, yr). Within a single system of measurement, express  measurements in a larger unit in terms of a smaller unit. A table of equivalencies will be  provided. Example 1: Know that 1 kg is 1,000 times as heavy as 1 g. Example 2:  Express the length of a 4-foot snake as 48 in.  **M04.D-M.1.1.2:** Use the four operations to solve word problems involving distances, intervals of time  (such as elapsed time), liquid volumes, masses of objects; money, including problems  involving simple fractions or decimals; and problems that require expressing  measurements given in a larger unit in terms of a smaller unit.  **M04.D-M.1.1.3:** Apply the area and perimeter formulas for rectangles in real-world and mathematical  problems (may include finding a missing side length). Whole numbers only. The  formulas will be provided.  **M04.D-M.1.1.4:** Identify time (analog or digital) as the amount of minutes before or after the hour.  Example 1: 2:50 is the same as 10 minutes before 3:00. Example 2: Quarter past six is  the same as 6:15. |
| **PA:CC.2.4.4.A.1:** Solve problems involving measurement and conversions from a larger unit to a smaller unit.  **Common Core: 4.MD.1**: Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...  **Unpacking:** The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, milliliters, and seconds. Students’ prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time. Students did not convert measurements. Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create. Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12.  Foundational understandings to help with measure concepts: •Understand that larger units can be subdivided into equivalent units (partition).  •Understand that the same unit can be repeated to determine the measure (iteration). •Understand the relationship between the size of a unit and the number of units needed (compensatory principal).  •Recognize that kilometers (km), meters (m) and centimeters (cm) are within the same system of metric units of length/distance.  •Identify objects/distances that are approximately 1 km, 1 m, and 1 cm.  •Recognize that a kilometer is 1000 times as long as a meter which is 100 times as long as a centimeter.  •Recognize that yards (yd), feet (ft), and inches (in) are within the same system of units of length/distance.  •Identify objects/distances that are approximately 1 yard, 1 foot, and 1 inch.  •Recognize that 1 yard is 3 feet, and 1 foot is 12 inches.  •Recognize that kilograms (kg) and grams (g) are within the same system of metric units of mass.  •Identify objects that are approximately 1 kg and 1 g.  •Recognize that a kilogram is 1000 times the mass of a gram.  •Recognize that pounds (lb) and ounces (oz) are within the same system of units of weight.  •Identify objects that are approximately 1 lb and 1 oz.  •Recognize that a pound is 16 times as heavy as an ounce.  •Recognize that liters (l) and milliliters (ml) are within the same system of units of liquid volume.  •Identify objects that hold approximately 1 liter and 1 ml.  •Recognize that a liter is 1000 times more than milliliter.  •Recognize that hours (hr), minutes (min) and seconds (sec) are within the same system of units of time.  •Measure time elapsed in an hour, minute, and second.  •Recognize that an hour is 60 minutes, and a minute is 60 seconds. •Record equivalent measurements within the same system of units in a table. | |

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| **M04.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.D-M.1 Solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit. (Page 2 of 2) | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft),**  **yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter** | |
| **DESCRIPTOR**  This EC addressed  on previous page.  **M04.D-M.1.1:** Solve problems involving length, weight (mass), liquid volume, time, area, and perimeter. | **ELIGIBLE CONTENT**  **M04.D-M.1.1.1:** Know relative sizes of measurement units within one system of units including standard  units (in., ft, yd, mi; oz., lb; c, pt, qt, gal), metric units (cm, m, km; g, kg; mL, L), and  time (sec, min, hr, day, wk, mo, yr). Within a single system of measurement, express  measurements in a larger unit in terms of a smaller unit. A table of equivalencies will be  provided. Example 1: Know that 1 kg is 1,000 times as heavy as 1 g. Example 2:  Express the length of a 4-foot snake as 48 in.  **M04.D-M.1.1.2:** Use the four operations to solve word problems involving distances, intervals of time  (such as elapsed time), liquid volumes, masses of objects; money, including problems  involving simple fractions or decimals; and problems that require expressing  measurements given in a larger unit in terms of a smaller unit.  **M04.D-M.1.1.3:** Apply the area and perimeter formulas for rectangles in real-world and mathematical  problems (may include finding a missing side length). Whole numbers only. The  formulas will be provided.  **M04.D-M.1.1.4:** Identify time (analog or digital) as the amount of minutes before or after the hour.  Example 1: 2:50 is the same as 10 minutes before 3:00. Example 2: Quarter past six is  the same as 6:15. |
| **PA:CC.2.4.4.A.1:** Solve problems involving measurement and conversions from a larger unit to a smaller unit.  **Common Core: 4.MD.2**: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.  **Unpacking:** When presented with word problems that involve distance, intervals of time, liquid volumes, masses of objects, or money:  • Solve the word problem by:  -Using any of the four operations;  -Measurements in whole numbers, simple fractions (with denominators 2, 3, 4, 5, 6, 8, 10, 12, & 100), or simple decimals (to hundredths);  -Converting larger measurements into smaller measurements within the same system of unit.  •Represent measurement quantities using diagrams (such as number line diagrams) that feature a measurement scale.  This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems.  **Common Core: 4.MD.3:** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.  **Unpacking**: Students developed understanding of area and perimeter in 3rd grade by using visual models. While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work. The formula for area is I x w and the answer will always be in square units. The formula for perimeter can be 2 l + 2 w or 2 (l + w) and the answer will be in linear units. This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization. Example: Mr. Rutherford is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will he need to cover the entire course?    NOTE: At Grade 4, in the “National” Common Core, there is no standard listed for time or money. | |

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| **M04.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.D-M.2 Represent and interpret data. | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: data, line plot, length, fractions** | |
| **DESCRIPTOR**  **M04.D-M.2.1:** Organize, display, and answer questions based on data. | **ELIGIBLE CONTENT**  **M04.D-M.2.1.1:** Make a line plot to display a data set of measurements in fractions of a unit (e.g.,  intervals of 1/2, 1/4, or 1/8).  **M04.D-M.2.1.2:** Solve problems involving addition and subtraction of fractions by using information  presented in line plots (line plots must be labeled with common denominators, such as  1/4, 2/4, 3/4).  **M04.D-M.2.1.3:** Translate information from one type of display to another (table, chart, bar graph, or  pictograph). |
| **PA:CC.2.4.4.A.2:** Translate information from one type of data display to another.  **PA:CC.2.4.4.A.4:** Represent and interpret data involving fractions using information provided in a line plot.  **Common Core: 4.MD.4:** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*  **Unpacking:** • Measure several objects (at least 10 objects is suggested) to the nearest eighth, quarter or half, and then  - Construct a line plot with:  ^A horizontal axis with tick marks or numbers that are evenly spaced out;  ^Enough tick marks or numbers to include the entire range of data;  ^At least two tick marks numbered (usually, at least the first and last tick mark be numbered, but all tick marks can be numbered,  or every other tick mark can be numbered);  ^The axis labeled with a name and the unit of measurement; and  ^X’s (or other symbol) that are a uniform size and vertically stacked uniformly.  This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot. Example: Students measured objects in their desk to the nearest **¼** , **½** , or 1/8 inch. They displayed their data collected on a line plot. How many object measured **½** inch? ¼ inch? If you put all the objects together end to end what would be the total length of all the objects. | |

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| **M04.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M04.D-M.3 Geometric measurement: understand concepts of angle; measure and create angles. | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decomposed, addition, subtraction, unknown** | |
| **DESCRIPTOR**  **M04.D-M.3.1:**Use appropriate tools and units to sketch an angle and determine angle measurements. | **ELIGIBLE CONTENT**  **M04.D-M.3.1.1:** Measure angles in whole-number degrees using a protractor. With the  aid of a protractor, sketch angles of specified measure.  **M04.D-M.3.1.2:** Solve addition and subtraction problems to find unknown angles on a  diagram in real-world and mathematical problems. (Angles must be  adjacent and non-overlapping.) |
| **PA:CC.2.4.4.A.6:** Measure angles and use properties of adjacent angles to solve problems.  **Common Core: 4.MD.6:** Measure angles in whole number degrees using a protractor. Sketch angles of specified measure.  **Unpacking:** Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).    **Common Core:4.MD.7:** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.  **Unpacking:** •Decompose an angle into two non-overlapping parts, and recognize that the angle measure of the whole angle is the sum of the angle measure of the parts. •Solve real world and mathematical problems that involve using addition and subtraction to find the unknown angle(s) on a diagram that shows a whole angle decomposed into non-overlapping angles with some angle measures given.  This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.    Example: A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved? If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?  Example**:** Joey knows that when a clock’s hands are exactly on 12 and 1, the angle formed by the clock’s hands measures30º. What is the measure of the angle formed when a clock’s hands are exactly on the 12 and 4?  **NOTE: Common Core:4.MD.5** does not have a specific correlation to the PA 4th grade common core standards. This standard, as stated in the “National” Common Core reads: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:  a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.  b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. The unpacking of this standard is:  •Draw an angle and identify its parts (vertex, rays).  •Explain what an angle is and what it measures.  •Recognize that a circle measures 360 degrees and a turn that is 1/360 of a circle is called a one-degree angle.  •Explain what it means for an angle to have a measure of *n* degrees.  This standard brings up a connection between angles and circular measurement (360 degrees). This standard calls for students to explore an angle as a series of “one-degree turns.” A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 degrees, how many  one-degree turns has the sprinkler made? | |

Table 1 Common addition and subtraction situations

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|  | **Result Unknown** | **Change Unknown** | **Start Unknown** |
| **Add to** | Two bunnies sat on the grass. Three  more bunnies hopped there. How many  bunnies are on the grass now?  2 + 3 = ? | Two bunnies were sitting on the grass.  Some more bunnies hopped there. Then  there were five bunnies. How many  bunnies hopped over to the first two?  2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?  ? + 3 = 5 |
| **Take From** | Five apples were on the table. I ate two  apples. How many apples are on the  table now?  5 – 2 = ? | Five apples were on the table. I ate  some apples. Then there were three  apples. How many apples did I eat?  5 – ? = 3 | Some apples were on the table. I ate two  apples. Then there were three apples.  How many apples were on the table  before? ? – 2 = 3 |
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|  | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown2** |
| **Put Together/ Take Apart3** | Three red apples and two green apples  are on the table. How many apples are  on the table?  3 + 2 = ? | Five apples are on the table. Three are  red and the rest are green. How many  apples are green?  3 + ? = 5, 5 – 3 = ? | Grandma has five flowers. How many  can she put in her red vase and how  many in her blue vase?  5 = 0 + 5, 5 = 5 + 0  5 = 1 + 4, 5 = 4 + 1  5 = 2 + 3, 5 = 3 + 2 |
|  |  |  |  |
|  | Difference Unknown | Bigger Unknown | Smaller Unknown |
| **Compare4** | (“How many more?” version):  Lucy has two apples. Julie has five  apples. How many more apples does  Julie have than Lucy?  (“How many fewer?” version):  Lucy has two apples. Julie has five  apples. How many fewer apples does  Lucy have than Julie?  2 + ? = 5, 5 – 2 = ? | (Version with “more”):  Julie has three more apples than Lucy.  Lucy has two apples. How many apples  does Julie have?  (Version with “fewer”):  Lucy has 3 fewer apples than Julie.  Lucy has two apples. How many apples  does Julie have?  2 + 3 = ?, 3 + 2 = ? | (Version with “more”):  Julie has three more apples than Lucy.  Julie has five apples. How many apples  does Lucy have?  (Version with “fewer”):  Lucy has 3 fewer apples than Julie.  Julie has five apples. How many apples  does Lucy have?  5 – 3 = ?, ? + 3 = 5 |

C

2These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

3Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

4For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2 Common multiplication and division situations1

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|  | **Unknown Product**  **3 x 6 = ?** | **Group Size Unknown**  **(“How many in each group?” Division)**  **3 x ? = 18 and 18 ÷ 3 = ?** | **Number of Groups Unknown**  **(“How many groups?” Division)**  **? x 6 = 18 and 18 ÷ 6 = ?** |
| **Equal Groups** | There are 3 bags with 6 plums in  each bag. How many plums are  there in all?  *Measurement example.* You need 3  lengths of string, each 6 inches  long. How much string will you  need altogether? | If 18 plums are shared equally into 3 bags,  then how many plums will be in each bag?  *Measurement example.* You have 18 inches  of string, which you will cut into 3 equal  pieces. How long will each piece of string  be? | If 18 plums are to be packed 6 to a  bag, then how many bags are needed?  *Measurement example.* You have 18  inches of string, which you will cut  into pieces that are 6 inches long. How  many pieces of string will you have? |
| **Arrays2 , Area3** | There are 3 rows of apples with 6  apples in each row. How many  apples are there?  *Area example.* What is the area of a  3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows,  how many apples will be in each row?  *Area example.* A rectangle has area 18  square centimeters. If one side is 3 cm long,  how long is a side next to it? | If 18 apples are arranged into equal  rows of 6 apples, how many rows will  there be?  *Area example.* A rectangle has area 18  square centimeters. If one side is 6 cm  long, how long is a side next to it? |
|  |  |  |  |
|  | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown2** |
| **Compare** | A blue hat costs $6. A red hat costs  3 times as much as the blue hat.  How much does the red hat cost?  *Measurement example.* A rubber  band is 6 cm long. How long will  the rubber band be when it is  stretched to be 3 times as long? | A red hat costs $18 and that is 3 times as  much as a blue hat costs. How much does a  blue hat cost?  *Measurement example.* A rubber band is  stretched to be 18 cm long and that is 3  times as long as it was at first. How long  was the rubber band at first? | A red hat costs $18 and a blue hat  costs $6. How many times as much  does the red hat cost as the blue hat?  *Measurement example.* A rubber band  was 6 cm long at first. Now it is  stretched to be 18 cm long. How many  times as long is the rubber band now as  it was at first? |
| **General** | a x b = ? | a x ? = p and p ÷ a = ? | ? x b = p and p ÷ b = ? |

Co

1The first examples in each cell are examples of discrete things. These are easier or students and should be given before the measurement examples.

2The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in

the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

3Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially

important measurement situations.

Table 3 The properties of operations.

Here *a, b* and *c* stand for arbitrary numbers in a given number system. The properties apply to the rational number system, the real number system, and the complex number system.

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| **Property** | **Example** |
| *Associative property of addition* | (a + b) + c = a + (b + c) |
| *Commutative property of addition* | a + b = b + a |
| *Additive identity property of 0* | a + 0 = 0 + a = a |
| *Associative property of multiplication* | (a x b) x c = a x (b x c) |
| *Commutative property of multiplication* | a x b = b x a |
| *Multiplicative identity property of 1* | a x 1 = 1 x a = a |
| *Distributive property of multiplication over addition* | a x (b + c) = a x b + a x c |